

MATHEMATICS SL TZ1

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 47	48 - 58	59 - 70	71 - 82	83 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2011 examination session the IB has produced time zone variants of the Mathematics SL papers.

General Comments

Many thanks to the teachers who provided feedback through the G2 forms about the examination. These were read by the senior examining team prior to setting the grade boundaries, and provided helpful and often insightful discussion points for consideration for this grade award and for future paper setting. Many of the issues raised regarding individual questions are contained within this report.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

The vast majority of the work presented came from the set of tasks developed by the IB. Most schools appeared to be aware of the requirement to use these new tasks for this session. A few schools presented older tasks that are no longer usable for the portfolio and candidates suffered a penalty as a result. A very few teachers presented tasks of their own design. These

varied in quality but included some very good ones. Others lacked the necessary depth for a portfolio task and did not allow for success against all levels of the criteria.

Candidate performance against each criterion

Criterion A

Overall, candidates and teachers are making a good effort to ensure the use of proper notation. However, despite many years of comments about the inappropriate use of computer and calculator notation, there persists a minority of schools that do not penalize these errors. Often moderators would note that a comment had been made on candidate work that these notations were inappropriate, yet no penalty was applied.

Moderators are also noting an increasing prevalence of informal language for mathematical terms and operations. One purpose of these tasks is to improve the standard of mathematical language and terminology use. Teachers should be on the lookout for confusion between “quadratic” and “exponential”, “curve” and “line”, “variable” and “parameter”, etc.

Criterion B

The vast majority of work presented is communicated well. Issues that persist include the inadequate or poor labelling of graphs, the use of a “question & answer” format, overly detailed descriptions of calculator steps, and the use of appendices for graphs and tables that should appear in the body of the work. In some tasks, for example the “Stellar Numbers” task, the use of suitable diagrams is not only recommended, but required. Often candidates made claims about how many dots appeared at a stage in the pattern without any supporting evidence in the form of a clear diagram.

Criterion C

Type I

While most candidates were successful in discovering appropriate patterns in these tasks, the resulting statements often came out of the blue, with little or no supporting analysis and examples. Teachers should take note that results presented without adequate support cannot be accepted. Once a statement is proposed the candidate must use new and further examples to validate the conjecture. Many use the same values that they used to develop the statement in the first place, which will obviously satisfy the statement.

Type II

There was an improvement noted in the quality of work presented regarding the definition and declaration of variables, constraints and parameters. However, many candidates give this point short shrift and leave much to be assumed. As with Type I tasks, there must be sufficient analysis present in order to accept the proposed model as a result. Teachers should be aware that the use of calculator regression techniques to develop a model function will limit the mark in criterion C to level 2. In some cases candidates would use regression to find a suitable model then work backwards to show some analysis that “leads” to this model. This is inappropriate and should be considered as if the regression were used alone.

In the “Population Trends in China” task many candidates used only a linear model. While the data certainly looks linear, candidates should realize that a linear model over the longer run is not likely to be appropriate. Other models should also be considered and developed.

A qualitative consideration of the fit of the function to the data is sufficient provided that there is some substance to the comments. Statements such as “it fits well” say little and are not enough to achieve level 4. There is no expectation that the error be measured in any way.

Applying the analytically developed model to a further set of data and commenting on how well the model fits this new data is sufficient for level 5. Candidates will make modifications to their model so that it does fit better and this is recognized under criterion D.

Criterion D

Type I

Many candidates obtain good results and present admirable arguments for allowing appropriate values or to explain the behaviour noted. However, the results obtained must come from some sound reasoning in order to achieve the higher levels of criterion D. A general statement that appears from nowhere cannot be considered more than an attempt at achieving what was desired. Some candidates continue to limit the discussion of scope and limitations to only the most superficial observations. While it may appear obvious that a certain value can only be, for example, a natural number, the candidate should check whether or not other values happen to work in the general statement and what this implies. Candidates also find it difficult to offer informal explanations for their statements. This may sometimes be an algebraic argument or it may simply be a clearly drawn series of diagrams indicating the progression of a geometric structure.

Type II

The most obvious weakness in criterion D was the lack of consideration of the actual context. Many candidates do an admirable job of the mathematical work but neglect to relate the graphs and functions back to the context of the task. A task about G-force should be discussed in terms of the forces on the human body under different circumstances, not just increasing or decreasing values of variables or asymptotic behaviour of graphs. The best work often included thoughtful consideration of why there might be an asymptote in the G-force model, or why there was a fairly abrupt change in the population trend in China.

Criterion E

The access and quality of technology available has increased to a point where its use has become commonplace. Unfortunately moderators find that teachers do not properly inform them about the availability of technology in the school. High marks were often given in criterion E without any substantive evidence in the work, nor in any background information. Even in Type I tasks technology can often be used to produce results for many more and larger values of the variables, or for presenting graphs that support the conjecture. In Type II tasks multiple graphs can be used to provide evidence of an evolution of transformations that lead to a better fitting function, or for comparing multiple functions at one time.

Criterion F

This criterion was generally well assessed. Most of the marks were appropriately awarded at level F1. Teachers are reminded that 0 and F are reserved for work at either extreme; totally unacceptable or highly remarkable.

Recommendations for the teaching of future candidates

Teachers must work through the tasks beforehand so they have a good idea of what is possible and what to expect from their students. They are then better prepared to assist students in their understanding of the criteria and how the students can address the highest levels. Older tasks that are no longer allowed for final submission can be used for practice in this regard. Integrating small parts of these tasks into lessons can draw attention to the skills and concepts at work. This is especially important when teaching students how to develop models analytically or validate conjectures properly. Teachers should take time to teach the effective use of any software that might be useful.

Reading this subject report can give teachers and students a clearer idea of what is expected and what to watch for.

Further comments

Wherever tasks are adapted or self-designed teachers should try to avoid extensions that seriously increase the workload expected from students. The extra work is often too much for students.

Where there is more than one teacher in a school, it is essential that they standardise their marking so as to ensure a consistent and appropriate approach. Teachers are also encouraged to become IA moderators themselves. In this way they can be exposed to work of differing standard that is done by candidates all over the world, learning along the way how to improve their own teaching.

External assessment

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 51	52 - 62	63 - 72	73 - 90

The areas of the programme and examination that appeared difficult for the candidates

- Expected value of a discrete probability distribution
- Discriminant of a quadratic
- Conditional probability and independence
- Graphical interpretations of derivatives

The areas of the programme and examination in which candidates appeared well prepared

Candidates were quite adept with straightforward questions and the mathematical manipulations. Making graphical interpretations of situations proved to be a weakness, as shown in questions 7b, 8b and 10c. Students also demonstrate difficulty in reasoning mathematically toward a given answer, as revealed by the number of candidates who worked backwards on “show that” questions.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

A majority of candidates found success in the opening question. Common errors in (a) were to give $f \circ g$ or to multiply f by g . For (b) some gave the inverse as the reciprocal function $\frac{1}{x+3}$, or wrote $x = y + 3$. Most candidates chose to find a composite in (c), sometimes making simple errors when working with brackets and a negative sign. Only a handful used the more efficient $f(2) = 3$. Additionally, it was not uncommon for candidates to give a correct substitution but not complete the result. Simple expressions such as $(7 - 2x) + 3$ should be finished as $10 - 2x$.

Question 2

Many candidates answered this question well. Some continue to write the vector equation in (a) using “ $L =$ ”, which does not earn full marks. Part (b) proved accessible for most, although small arithmetic errors were not uncommon. Some candidates substituted $t = 2$ into the

original equation, and a few answered $\vec{OP} = \begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}$. A small but surprising number of

candidates left this question blank, suggesting the topic was not given adequate attention in course preparation.

Question 3

Also accessible, many candidates answered this question completely correct. Some found a determinant of 10 in part (a), but still earned full marks in follow-through. For (b), a common error was to right-multiply by A^{-1} . Furthermore, rather than using the inverse found in (a), some attempted to create a system of equations in four variables to solve the equation. Few were successful in this approach.

Question 4

Although many candidates were successful in working with the probability function, students had difficulty following the “show that” instruction of this question. Many substituted $k = 3$ and worked backwards to show that the sum of probabilities is 1. Some would argue that $k = 4$ does not work, but were unable to give a complete justification for $k = 3$. A good number of students seemed unprepared to find an expected value. Many candidates wrote a formula and did not know what to do with it, while others divided $E(X)$ by 3 or by 6, which

confuses the concept of a mean in a probability distribution with the more common understanding.

Question 5

Many candidates clearly knew their quotient rule, although a common error was to simplify $2x \ln x$ as $2 \ln x^2$ and then “cancel” the exponents. For (b), those who knew to set the derivative to zero typically went on to find the correct x -coordinate, which must be in terms of e , as this is the calculator-free paper. Occasionally, students would take $\frac{1-2 \ln x}{x^3} = 0$ and attempt to solve from $1 - 2 \ln x = x^3$.

Question 6

By far the most common error was to “cancel” the $\cos x$ and find only two of the three solutions. It was disappointing how few candidates solved this by setting factors equal to zero. Some candidates wrote all three answers from $\sin x = 1$, which only earned two of the three final marks. On a brighter note, many candidates found the $\frac{5\pi}{2}$, which showed an appreciation for the period of the function as well as the domain restriction. A handful of candidates cleverly sketched both graphs and used the intersections to find the three solutions.

Question 7

Those who knew to set the discriminant to zero had little trouble completing part (a). Some knew that having two equal roots means the factors must be the same, and thus surmised that $k = \frac{1}{2}$ will achieve $(x-1)(x-1)$. This is a valid approach, provided the reasoning is completely communicated. Many candidates set $f = 0$ and used the quadratic formula, which misses the approach entirely.

Part (b) proved challenging for most, and was often left blank. Those who considered a graphical interpretation and sketched the parabola found greater success.

Question 8

A majority of candidates found the values in the Venn diagram easily. Common errors include giving $s = 16$, and also neglecting s in finding $q = 4$ (e.g. $12 + 8 - 16$). Some interpreted the values as probabilities, despite the question explicitly stating that p , q , r and s represent numbers of students. Occasionally the values for p and r were misinterpreted as being inclusive of q . Follow-through marks were often earned in subsequent parts for such cases.

For (b), rather than think of the situations conceptually, most candidates reached for the formula for conditional probability, with mixed results. Few candidates considered that independence means $P(A/M) = P(A)$. Most applied $P(A \cap B) = P(A) \times P(B)$, with many giving incomplete or incorrect calculations. Some candidates compared the wrong things and showed, for example, that $\frac{5}{8} \neq \frac{3}{8}$, which incorrectly compares $P(A/M)$ with $P(A \cap M)$.

Others stated that because there is an intersection, the events are independent, which is an insufficient explanation.

Part (c) was commonly answered as if there is replacement, with many candidates calculating $\frac{3}{16} \times \frac{7}{16}$. However, implicit in the phrasing “one after the other” is that there is no replacement.

Question 9

Many candidates answered (a) correctly, although some reversed the vectors when finding \vec{BC} , while others miscopied the vectors from the question paper. Students had no difficulty finding the scalar product and magnitudes of the vectors used in finding the cosine. However, few recognized that \vec{BA} is the vector to apply in the formula to find the cosine value. Most used \vec{AB} to obtain a positive cosine, which neglects that the angle is obtuse and thus has a negative cosine. Surprisingly few students could then take a value for cosine and use it to find a value for sine. Most left (bii) blank entirely.

Part (c) proved accessible for many candidates. Some created an expression for $|\vec{CD}|$ and then substituted the given $p = 3$ to obtain $\sqrt{50}$, which does not satisfy the “show that” instruction. Many students recognized that the scalar product must be zero for vectors to be perpendicular, and most provided the supporting calculations.

Question 10

Many candidates gave a correct initial velocity, although a substantial number of candidates answered that $0 + \cos 0 = 0$. For (b), students commonly applied the chain rule correctly to achieve the derivative, and many recognized that the acceleration must be zero. Occasionally a student would use a double-angle identity on the velocity function before differentiating. This is not incorrect, but it usually caused problems when trying to show $k = \frac{\pi}{4}$. At times students

would reach the equation $\sin 2k = 1$ and then substitute the $\frac{\pi}{4}$, which does not satisfy the “show that” instruction.

The challenge in this question is sketching the graph using the information achieved and provided. This requires students to make graphical interpretations, and as typical in section B, to link the early parts of the question with later parts. Part (a) provides the y -intercept, and part (b) gives a point with a horizontal tangent. Plotting these points first was a helpful strategy. Few understood either the notation or the concept that the function had to be increasing on either side of the $\frac{\pi}{4}$, with most thinking that the point was either a max or min. It was the astute student who recognized that the derivatives being positive on either side of $\frac{\pi}{4}$ creates a point of inflexion.

Additionally, important points should be labeled in a sketch. Indicating the $\frac{\pi}{4}$ on the x -axis is a requirement of a clear graph. Although students were not penalized for not labeling the $\frac{\pi}{2}$ on the y -axis, there should be a recognition that the point is higher than the y -intercept.

While some candidates recognized that the distance is the area under the velocity graph, surprisingly few included neither the limits of integration in their expression, nor the “ dt ”. Most unnecessarily attempted to integrate the function, often giving an answer with “ $+C$ ”, and only

earned marks if the limits were included with their result. Few recognized that a shaded area is an adequate representation of distance on the sketch, with most fruitlessly attempting to graph a new curve.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 33	34 - 44	45 - 54	55 - 65	66 - 75	76 - 90

The areas of the programme and examination that appeared difficult for the candidates

Candidates in this session had difficulties in the following areas of the programme:

- Normal distributions
- Binomial distributions
- Logarithmic functions
- Area between two curves
- Second derivative test and points of inflection
- Trigonometric modelling
- Graphs of inverse functions
- Recognizing a binomial experiment
- Appropriate and timely use of a GDC

The areas of the programme and examination in which candidates appeared well prepared

For students who were well prepared, there was ample opportunity to demonstrate a high level of knowledge and understanding on this paper. The following areas of the programme were handled well by most students:

- Solutions of triangles
- Arithmetic series
- Vertex form of a quadratic function
- Simple probability, combined events and tree diagrams
- First and second derivatives
- Basic logarithmic manipulation

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Solutions of triangles

The majority of candidates were successful with this question. Most correctly used the cosine rule in part (a) and the sine rule in part (b). Some candidates did not check that their GDC was set in degree mode while others treated the triangle as if it were right angled. A large number of candidates were penalized for not leaving their answers exactly or to three significant figures.

Question 2: Transformations of quadratic functions

Most candidates had little difficulty with this question. In part (c), a few reflected the vertex in the y – axis rather than the x – axis.

Question 3: Arithmetic sequences

The majority of candidates gained full marks in this question. However, the presentation of work was often disappointing, and there were a few incorrect trial and error approaches in part (a) resulting in division by 20 rather than 19.

Question 4: Normal distributions

This question clearly demonstrated that some centres are still not giving adequate treatment to this topic. A great many candidates neglected to find the standard deviation and used the variance throughout. More still did not leave their answers to the required accuracy. Ignoring the use of the variance, responses to part (a) demonstrated that most candidates were comfortable finding the required probability using their calculator or setting up a suitable standardized equation. In part (b) (i), the sketch was often poorly shaded or incorrectly labelled. In (b) (ii), candidates frequently confused the z -score with the given probability of 0.85. Calculator approaches were more successful than working by hand but candidates should remember to avoid the use of calculator notation in their working, as it is not correct mathematical notation.

Question 5: Binomial distribution

Part (a) was answered correctly by most candidates. In parts (b) and (c), many failed to recognize the binomial nature of this experiment and opted for incorrect techniques in simple probability. Although several candidates appreciated that (c) involved the idea of a complement, some resorted to elaborate probability addition suggesting they were unaware of the capabilities of their GDC. There was also a great deal of evidence to suggest that candidates did not understand the phrase “at least 10” as several candidates found either $1 - P(X \leq 10)$, $1 - P(X = 10)$ or $P(X > 10)$.

Question 6: Area between curves

This question was poorly done by a great many candidates. Most seemed not to understand what was meant by the phrase ‘region enclosed by’ as several candidates assumed that the limits of the integral were those given in the domain. Few realized what area was required, or that intersection points were needed. Candidates who used their GDCs to first draw a suitable sketch could normally recognize the required region and could find the intersection points correctly. However, it was disappointing to see the number of candidates who could not then use their GDC to find the required area or who attempted unsuccessful analytical approaches.

Question 7: Combined events

Part (a) was usually well done. Those candidates that did not succeed with this part often did not show a correct tree diagram indicating that they did not really understand the problem or indeed how to start it. Many successful attempts to (b) relied on 'guess and check' or intuitive solutions while a surprising number of candidates could not manage to systematically set up an appropriate algebraic expression involving a complement.

Question 8: Trig functions

Parts (a) and (b) were generally well done, however there were several instances of candidates working backwards from the given answer in part (b). Parts (c) and (d) proved to be quite challenging for a large proportion of candidates. Many did not attempt these parts. The most common error was a misinterpretation of the word "descending" where numerous candidates took $h'(t)$ to be 0.5 instead of -0.5 but incorrect derivatives for h were also

widespread. The process required to solve for t from the equation $-0.5 = \frac{4\pi}{15} \cos\left(\frac{\pi t}{15}\right)$

overwhelmed those who attempted algebraic methods. Few could obtain both correct solutions, more had one correct while others included unreasonable values including $t < 0$. In part (d), not many understood that the condition for underwater was $h(t) < 0$ and had trouble interpreting the meaning of "second value". Many candidates, however, did recover to gain some marks in follow through.

Question 9: Points of inflexion

Most candidates were able to recognize the points of inflexion in part (a) and had little difficulty with the first and second derivatives in part (b). A few did not recognize the application of the product rule in part (b). Obtaining the x -coordinates of the inflexion points in (c) usually did not cause many problems, however, only the better-prepared candidates understood how to set up a second derivative test in part (d). Many of those did not show, or clearly indicate, the values of x used to test for a point of inflexion, but merely gave an indication of the sign. Some candidates simply resorted to showing that $f''\left(\pm\frac{1}{\sqrt{2}}\right) = 0$,

completely missing the point of the question. The necessary condition for a point of inflexion, i.e. $f''(x) = 0$ **and** the change of sign for $f''(x)$, seemed not to be known by the vast majority of candidates.

Question 10: Logarithmic functions

Few candidates had difficulty with part (a) although it was often communicated using some very sloppy applications of the rules of logarithm – writing $\frac{\log 16}{\log 4}$ instead of $\log\left(\frac{16}{4}\right)$. Part

(b) was generally done well as was (c) (i) where candidates seemed quite comfortable changing bases. There were some very good sketches in (c) (ii), but there were also some very poor ones with candidates only considering shape and not the location of the x -intercept or the asymptote. A surprising number of candidates did not use the scale required by the question and/or did not use graph paper to sketch the graph. In some cases, it was evident that students simply transposed their graphs from their GDC without any analytical consideration. Part (d) was poorly done as candidates did not consider the command term,

“write down” and often proceeded to find the inverse function before making the appropriate substitution. Part (e) eluded a great many candidates as most preferred to attempt to find the inverse analytically rather than simply reflecting the graph of f in the line $y = x$. This graph also suffered from the same sort of problems as the graph in (c) (ii). Some students did not have their curve passing through (2, 4.5) nor did they clearly indicate its position as instructed. This point was often mislabelled on the graph of f . The efforts in this question demonstrated that students often work tenuously from one question to the next, without considering the “big picture”, thereby failing to make important links with earlier parts of the question

Recommendations and guidance for the teaching of future candidates for both papers

Teachers need to be sure their students are exposed to all areas of the syllabus. It was apparent that this is not always the case, as some candidates left questions blank or gave answers which made no sense. Too often it is clear that candidates are not given complete preparation in the areas of vectors and probability. It should be noted that the recommended teaching hours for probability and statistics is substantial and near equal to that of calculus.

It is also helpful to candidates if they can be familiar with the information booklet. However, it is not enough to simply know these formulas. Candidates need to know what kind of situations these formulas are used for. Then they also need to know what the values they are using represent, and how to manipulate and work with these formulas correctly.

Practicing exam-style questions under timed conditions can be helpful to candidates. While most candidates seemed to be able to finish the exam, there were many who seemed to be rushed at the end, and some who left the last parts blank, presumably because they ran out of time. Candidates need to understand that they should not need to spend a lot of time on a 1 or 2 mark question, and that a 9-mark question generally takes more time and requires more working to be shown. It would also be helpful if candidates could work on practice exam papers and then reflect on what they had done by looking at the requirements of the different command terms and at their use of time relative to the amount of marks for each question.

Some teachers expressed concern that some questions seemed to have too many marks allocated. During paper setting, marks are carefully allocated to questions based upon the amount of work needed for solution. Students should be encouraged to show full working, as an incorrect answer with complete working can earn the majority of the marks

Candidates should be familiar with the command terms and understand what is required. The command term “show that” is not well understood by many candidates. As this is not an obvious instruction, it is helpful if they are exposed to the terminology throughout the two years of the course, so as to become accustomed to its meaning.

Some candidates do not appear aware of the three significant figure requirement; this requires continued emphasis during the course.

Teachers should remind candidates that it is important to use proper notation throughout their working, as this makes their working easier to understand. It has often been noted by examiners that the stronger candidates tend to work through questions in a more organized manner. Poor mathematical communication can cause problems for candidates at this level. Teachers are encouraged to persevere with candidates emphasizing appropriate language

and set up of solutions. Avoid calculator language and notation when communicating solutions and encourage candidates to label questions and their parts.

Teachers should be encouraged to provide more opportunities for students to develop the quality of their explanations and justifications of important mathematical results. Design the course in such a way as to provide adequate time for students to develop conceptual understanding in conjunction with good technique. Encourage understanding through reading and communicating appropriate mathematical language. Expose students to more mathematics set in both familiar and unfamiliar contexts particularly in the areas of trigonometry and calculus.

In vector problems, candidates should develop an understanding of the techniques and should be encouraged to clearly indicate which vectors they are using when finding the angle between two lines.

Teachers are encouraged to ensure that candidates are familiar with all the GDC skills and techniques that are found in the guide and the GDC TSM. This can be achieved by incorporating the GDC into daily lessons to augment understanding of most syllabus topics. Candidates should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviours of functions.

Candidates need to be aware that not all equations can be solved using algebra; they will be expected to use their GDC to solve equations on Paper 2. They also need to be aware that a graph sketch or setting the equation equal to zero is suitable working for a GDC solution. Candidates should understand how to sketch an accurate graph from a GDC screen by using key graph features and/or the table function.

Unless otherwise specified, trigonometry questions are in radians. It was clear from teacher comments that some candidates were not aware of the importance of checking the mode of their calculator.

Many students seem to be formula-driven, and consequently, they have difficulty interpreting or explaining a situation. If teachers focus on concepts to develop methods of solution, then students will have greater success interpreting questions that introduce different situations.

With regards to e-marking, candidates and teachers need to be aware that **everything** on a scanned script will show up as dark black. This means that stray marks, ink from pens that bleed through the paper, and even items which have been partially erased will all show up as black when they are scanned. This often makes it difficult to decipher the candidates' intended working and answers. Candidates are reminded that graph paper should not be used for anything except the drawing of graphs, and that when a question uses the command term "sketch", it is generally not necessary to use graph paper.

Finally, many teachers are doing a very good job of preparing their students, and are encouraged to continue doing so. It is hoped that the comments here will help identify where there are weaknesses, and provide advice for future improvement.